Homework 1

Question 1

A typical input variable is identified for each of the following examples of dynamic systems. Give at least one output variable for each system.

1. **Human Body**: neuro-electric pulses.
2. **Company**: information.
3. **Power Plant**: fuel rated.
4. **Automobile**: steering wheel movement.
5. **Robot**: applied voltage to joint motor.
6. **Highway Bridge**: vehicle force.

Also, indicate other possible inputs.

1. Muscle contraction, body movement, body temperature, hate rate.

2. Decisions, proofits, products

3. Electric power, profit, customer satisfation pollution rate

4. Front wheel turn, direction of heading, noise level, accident, pollution level

5. Motions of joints and end-effector, trajoctory error, collision,

6. Bridge oscillation, overload, noise

Question 2

Give four categories of uses of dynamic modeling. List advantages and disadvantages of experimental modeling over analytical modeling.

Categories of uses of dynamic modeling:

system analysis, simulation and evaluation

system design and modification

fault detection and diagnosis

system control

Advantages: A real physical system

Realistic practical inputs

Feasible / practical system

Disadvantages: A physical / prototype is needed.

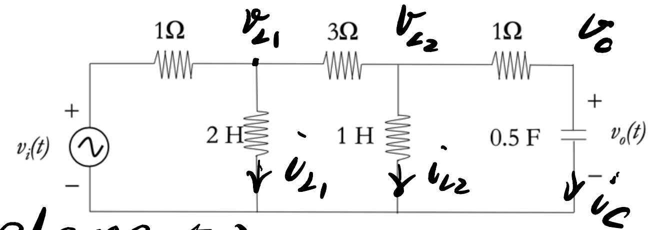
Test instrumentation and procedure can be costly.

Testing is slow

system modification can be difficult.

Question 3

Represent the electrical network in state space where $v_0(t)$ is the output.



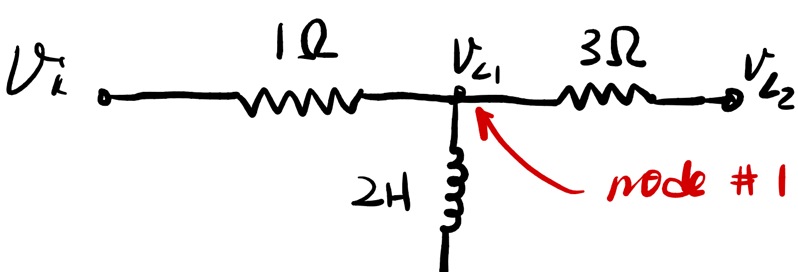
write the differential equations for each inductor and capacitor (energy storage element)

$$\begin{cases} v_{L1} = 2 \cdot \dfrac{di_{L1}}{dt} & (1) \\ v_{L2} = \dfrac{di_{L2}}{dt} & (2) \\ i_C = 0.5 \cdot \dfrac{dv_C}{dt} & (3) \end{cases}$$

Select i_{L1}, i_{L2} and V_C as the state variables and re-write the equations:

$$\begin{cases} \dot{i}_{L1} = \frac{1}{2} V_{L1} & (4) \\ \dot{i}_{L2} = V_{L2} & (5) \\ \dot{v}_C = 2 i_C & (6) \end{cases}$$

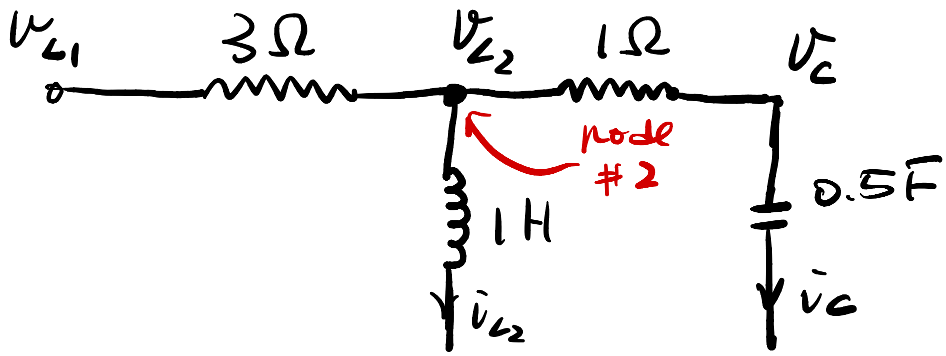
consider the node #1 below:



write the node equation for node #1

$$\frac{v_i - v_{L1}}{1} = i_{L1} + \frac{v_{L1} - v_{L2}}{3} \qquad (7)$$

Consider the node #2.



write the node equation for node #2.

$$\frac{V_{L1}-V_{L2}}{3}=\frac{V_{L2}-V_C}{1}+i_{L2} \qquad (8)$$

pay attention to the capacitor, we have another equation below:

$$\frac{V_{L2}-V_C}{1}=i_C \qquad (9)$$

put equations (7) (8) (9) together, we have.

$$\begin{cases} V_i - V_{L1} = i_{L1} + \frac{1}{3}V_{L1} - \frac{1}{3}V_{L2} \\ \frac{V_{L1}}{3} - \frac{V_{L2}}{3} = V_{L2} - V_C + i_{L2} \\ V_{L2} - V_C = i_C \end{cases}$$

Simplify the above equations, we get

$$\begin{cases} -\frac{4}{3}v_{L1} + \frac{1}{3}v_{L2} = i_{L1} - v_i & (10) \\ \frac{1}{3}v_{L1} - \frac{4}{3}v_{L2} = i_{L2} - v_C & (11) \\ i_C = v_{L2} - v_C & (12) \end{cases}$$

Base on Eq. (10) and (11), we solve for v_{L1} & v_{L2}:

$$\begin{cases} v_{L1} = -\frac{4}{5}i_{L1} - \frac{1}{5}i_{L2} + \frac{1}{5}v_C + \frac{4}{5}v_i & (13) \\ v_{L2} = -\frac{1}{5}i_{L1} - \frac{4}{5}i_{L2} + \frac{4}{5}v_C + \frac{1}{5}v_i & (14) \end{cases}$$

Substitute Eq (14) into Eq. (12), we get:

$$i_C = -\frac{1}{5}i_{L1} - \frac{4}{5}i_{L2} - \frac{1}{5}v_C + \frac{1}{5}v_i \quad (15)$$

Substitute Eq. (13) (14) (15) into Eq. (4) (5) (6), we have

$$\begin{cases} \dot{i}_{L1} = -\frac{2}{5}i_{L1} - \frac{1}{10}i_{L2} + \frac{1}{10}v_C + \frac{2}{5}v_i \\ \dot{i}_{L2} = -\frac{1}{5}i_{L1} - \frac{4}{5}i_{L2} + \frac{4}{5}v_C + \frac{1}{5}v_i \\ \dot{v}_C = -\frac{2}{5}i_{L1} - \frac{8}{5}i_{L2} - \frac{2}{5}v_C + \frac{2}{5}v_i \end{cases}$$

The vector-matrix form of the state equation:

$$\begin{bmatrix} \dot{i}_{L1} \\ \dot{i}_{L2} \\ \dot{v}_o \end{bmatrix} = \begin{bmatrix} -\frac{2}{5} & -\frac{1}{10} & \frac{1}{10} \\ -\frac{1}{5} & -\frac{4}{5} & \frac{4}{5} \\ -\frac{2}{5} & -\frac{8}{5} & -\frac{2}{5} \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ V_c \end{bmatrix} + \begin{bmatrix} \frac{2}{5} \\ \frac{1}{5} \\ \frac{2}{5} \end{bmatrix} v_i$$

Since the system output $V_o = V_c$

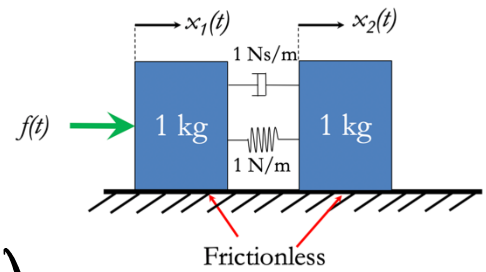
$$V_o = 0 \cdot i_{L1} + 0 \cdot i_{L2} + V_c + 0 \cdot V_i$$

the vector-matrix form of the output equation is:

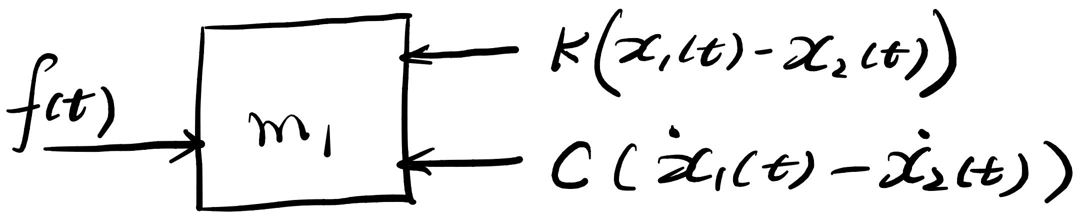
$$V_o = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ V_c \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} \cdot V_i$$

Question 4

Please find the state space model for the mechanical system below, assuming $f(t)$ as input, $x_1(t)$ and $x_2(t)$ as the outputs.



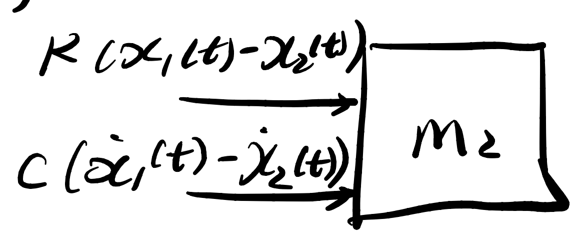
FBD of M_1



Therefore: $\Sigma F = ma \Rightarrow$

$$f(t) - K(x_1(t)-x_2(t)) - C(\dot{x}_1(t)-\dot{x}_2(t)) = m_1 \ddot{x}_1(t)$$

FBD of M_2



therefore:

$$K(x_1(t)-x_2(t)) + C((\dot{x}_1(t)-\dot{x}_2(t)) = m_2 \ddot{x}_2(t)$$

$$\ddot{x}_1 = -\frac{K}{m_1}(x_1(t)-x_2(t)) - \frac{C}{m_1}(\dot{x}_1(t)-\dot{x}_2(t)) + \frac{f(t)}{m_1}$$

$$\ddot{x}_2 = \frac{K}{m_2}(x_1(t)-x_2(t)); + \frac{C}{m_2}(\dot{x}_1(t)-\dot{x}_2(t))$$

To form state equation

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \ddot{x}_1(t) \\ \ddot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K}{m_1} & \frac{K}{m_1} & -\frac{C}{m_1} & \frac{C}{m_1} \\ \frac{K}{m_2} & -\frac{K}{m_2} & \frac{C}{m_2} & -\frac{C}{m_2} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_1} \\ 0 \end{bmatrix} f(t)$$

Substitute $m_1=1$, $m_2=1$, $C=1$ & $K=1$ into the Eq. above:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \ddot{x}_1(t) \\ \ddot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} f(t)$$

Since the system outputs are x_1 and x_2

$\Rightarrow$ $y_1 = x_1$ $\quad y_2 = x_2$

The output equation is

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + [0] f$$